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# MATHCOUNTS®

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2005

■ Chapter Competition ■  
Sprint Round  
Problems 1–30

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Name \_\_\_\_\_

School \_\_\_\_\_

**DO NOT BEGIN UNTIL YOU ARE  
INSTRUCTED TO DO SO.**

This round of the competition consists of 30 problems. You will have 40 minutes to complete the problems. You are not allowed to use calculators, books or any other aids during this round. If you are wearing a calculator wrist watch, please give it to your proctor now. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the right-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

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Total Correct	Scorer's Initials

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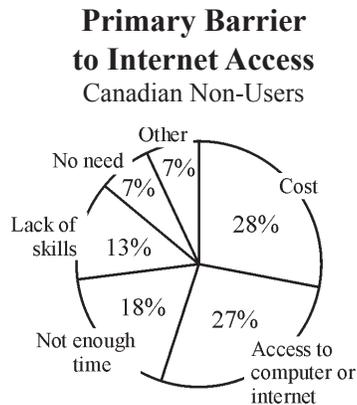
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1. The chart below gives the air distance in miles between selected world cities. What is the distance from Honolulu to Cape Town?

	Bangkok	Cape Town	Honolulu	London
Bangkok		6300	6609	5944
Cape Town	6300		11,535	5989
Honolulu	6609	11,535		7240
London	5944	5989	7240	

1. \_\_\_\_\_ miles

2. The pie chart shown represents a survey of Canadians who do not use the Internet. What is the percent of Canadian non-users for whom cost is *not* the primary barrier?



2. \_\_\_\_\_ %

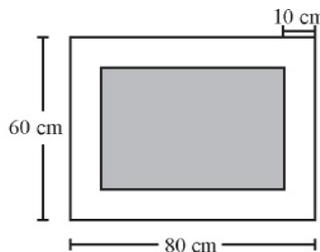
3. It is now 12:00:00 midnight, as read on a 12-hour digital clock. In 122 hours, 39 minutes and 44 seconds the time will be A:B:C. What is the value of  $A + B + C$ ?

3. \_\_\_\_\_

4. Shooting hoops for 30 minutes burns 150 calories. How many calories would Kendra burn shooting hoops if she shot hoops 30 minutes every day for one week?

4. \_\_\_\_\_ calories

5. Olga purchases a rectangular mirror (the shaded region) that fits exactly inside a frame. The outer perimeter of the frame measures 60 cm by 80 cm. The width of each side of the frame is 10 cm. What is the area of the mirror?



5. \_\_\_\_\_ sq cm

6. The product of two positive whole numbers is 2005. If neither number is 1, what is the sum of the two numbers?

6. \_\_\_\_\_

7. Three friends each ordered a large cheese pizza. Shauntee ate  $\frac{2}{3}$  of her pizza, Carlos ate  $\frac{8}{9}$  of his pizza and Rocco ate  $\frac{26}{27}$  of his pizza. If the remaining portions from the three pizzas are put together, what fraction of a large pizza do they make? Express your answer as a common fraction.

7. \_\_\_\_\_

8. It takes 24 minutes for Jana to walk one mile. At that rate, how far will she walk in 10 minutes? Express your answer as a decimal to the nearest tenth.



8. \_\_\_\_\_ miles

9. Roslyn has ten boxes. Five of the boxes contain pencils, four of the boxes contain pens, and two of the boxes contain both pens and pencils. How many boxes contain neither pens nor pencils?

9. \_\_\_\_\_ boxes



10. How many combinations of pennies, nickels and/or dimes are there with a total value of 25¢?



10. \_\_\_\_\_ combinations

11. What is the value of the following expression:  
 $1 - 3 + 5 - 7 + 9 - \dots - 43 + 45 - 47 + 49$ ?

11. \_\_\_\_\_

12. A rectangular tile measures 3 inches by 4 inches. What is the fewest number of these tiles that are needed to completely cover a rectangular region that is 2 feet by 5 feet?

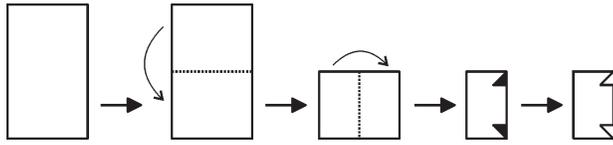
12. \_\_\_\_\_ tiles

13. When plotted in the standard rectangular coordinate system, trapezoid ABCD has vertices A(1, -2), B(1, 1), C(5, 7) and D(5, 1). What is the area of trapezoid ABCD?

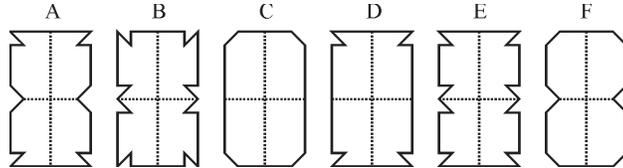
13. \_\_\_\_\_ square units

14. Trey receives a 5% commission on every sale he makes. On the sale of a \$60 coat (before any discounts), how many more cents will he receive if his commission is based on the original price of the coat rather than the price of the coat after a 20% discount? 14. \_\_\_\_\_ cents
15. Five distinct points A, B, C, D and E lie on a line, but not necessarily in that order. Use the information below to determine the number of units in the length of segment DC.
- E is the midpoint of segment AB.
  - D is the midpoint of segment AE.
  - Both C and E are the same distance from B.
  - The distance from D to B is 9 units.
15. \_\_\_\_\_ units
16. In the 19<sup>th</sup> century, Britain used a money system which included pence, farthings, shillings and pounds. The following conversions were used:
- 4 farthings = 1 pence
  - 12 pence = 1 shilling
  - 20 shillings = 1 pound
- How many total farthings were equivalent to 1 pound and 5 pence? 16. \_\_\_\_\_ farthings
17. What is the sum of all the distinct positive two-digit factors of 144? 17. \_\_\_\_\_
18. The points B(1, 1), I(2, 4) and G(5, 1) are plotted in the standard rectangular coordinate system to form triangle BIG. Triangle BIG is translated five units to the left and two units upward to triangle B'I'G', in such a way that B' is the image of B, I' is the image of I, and G' is the image of G. What is the midpoint of segment B'G'? Express your answer as an ordered pair. 18. \_\_\_\_\_ (   ,   )
19. The positive difference of the cube of an integer and the square of the same integer is 100. What is the integer? 19. \_\_\_\_\_

20. A rectangular sheet of paper is folded twice and then cut, as shown below. All fold lines are dashed, and the portion that is to be cut away is shaded.



Which of the following drawings (A, B, C, D, E or F) shows what the paper looks like when it is unfolded after the cuts?



20. \_\_\_\_\_

21. Henry took five tests, and his average score was 57 points. He scored at least 50 points on each test. There were 100 points possible on each test. What is the highest score that Henry could have earned on any of the five tests?

21. \_\_\_\_\_ points

22. How many ordered pairs  $(x, y)$  satisfy BOTH conditions below?  
 Condition I:  $x = 1$  or  $y = 0$  or  $y = 2$   
 Condition II:  $x = 0$  or  $x = 2$  or  $y = 1$

22. \_\_\_\_\_ ordered pairs

23. Zan has created this rule for generating sequences of whole numbers.

If a number is 25 or less, double the number.

If a number is more than 25, subtract 12 from it.

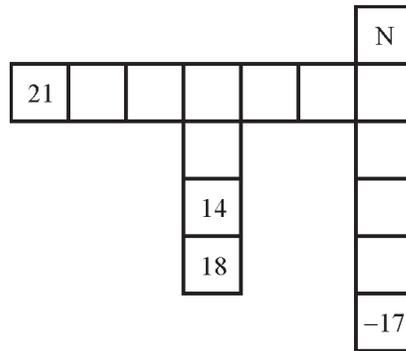
For example, if Zan starts with 10, she gets the sequence 10, 20, 40, 28, 16, ... . If the third number in Zan's sequence is 36, what is the sum of the four distinct numbers that could have been the first number in her sequence?

23. \_\_\_\_\_

24. Reverse the two digits of my age, divide by three, add 20, and the result is my age. How many years old am I?

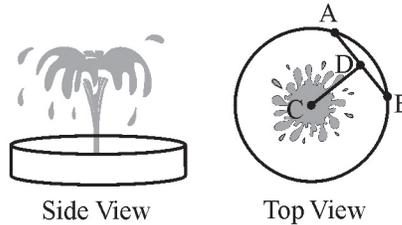
24. \_\_\_\_\_ years

25. The sequence of integers in the row of squares and in each of the two columns of squares form three distinct arithmetic sequences. What is the value of  $N$ ?



25. \_\_\_\_\_

26. To be able to walk to the center  $C$  of a circular fountain, a repair crew places a 16-foot plank from  $A$  to  $B$  and then a 10-foot plank from  $D$  to  $C$ , where  $D$  is the midpoint of  $\overline{AB}$ . What is the area of the circular base of the fountain? Express your answer in terms of  $\pi$ .



26. \_\_\_\_\_ square feet

27. The function  $f$  is defined by  $f(n) = f(n - 1) + f(n - 2)$ . It is also true that  $f(1) = 3$  and  $f(3) = 10$ . What is the value of  $f(6)$ ?

27. \_\_\_\_\_

28. If  $\frac{a}{b} = \frac{3}{4}$ ,  $\frac{b}{c} = \frac{8}{9}$  and  $\frac{c}{d} = \frac{2}{3}$ , what is the value of  $\frac{ad}{b^2}$ ? Express your answer as a common fraction.

28. \_\_\_\_\_

29. A play has two male roles, two female roles and two roles that can be either gender. Only a man can be assigned to a male role, and only a woman can be assigned to a female role. If five men and six women audition, in how many ways can the six roles be assigned?

29. \_\_\_\_\_ ways

30. What is the arithmetic mean of all of the positive two-digit integers with the property that the integer is equal to the sum of its first digit plus its second digit plus the product of its two digits?

30. \_\_\_\_\_