Geometric shapes occur in many places. After studying them using transformations, students start to see certain characteristics of different shapes. In these lessons students look at shapes more closely, noticing similarities and differences. They begin to classify them using Venn diagrams. Students begin to see the need for precise names, which expands their geometric vocabulary. The last lesson introduces probability.

See the Math Notes box in Lesson 1.3.1 for more information about Venn diagrams.

Example 1

Decide which shapes from the Lesson 1.2.6 Resource Page belong in each section of the Venn diagram below.
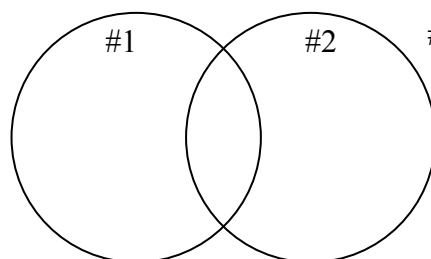
#1: Two pairs of parallel sides   #1   #2   #2: All sides are the same length

Circle #1, represents all shapes on the resource page that have two pairs of parallel sides. There are four figures on the resource page that have this characteristic: the rectangle, the square, the rhombus, and the parallelogram. These shapes will be contained in circle #1. Circle #2 represents all shapes on the resource page that have sides all the same length. There are five figures with this characteristic: the regular hexagon, the equilateral triangle, the square, the rhombus, and the regular pentagon. All five of these shapes will be completely contained in Circle #2. There are two shapes that are on both lists: the square and the rhombus. These two shapes have all sides the same length *and* they have two pairs of parallel sides. These two shapes, the square and the rhombus, must be listed in the region that is in both circles, which is shaded above. All other shapes would be placed outside the circles.
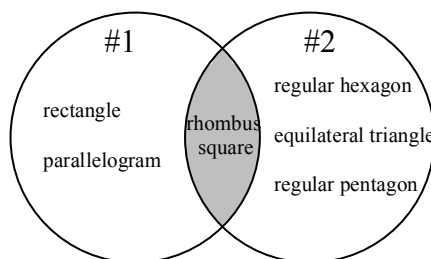
Example 2

Based on the markings of each shape below, describe the shape using the most specific name possible.

a.

b.

c.
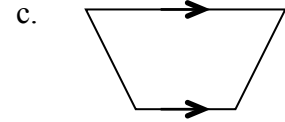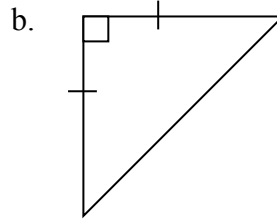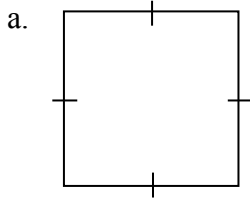
In Lesson 1.3.2 students created a Shapes Toolkit, that is, a resource page showing many different shapes. Using terms, definitions, and characteristics they had identified, students described the shapes on the resource page and added appropriate markings. Certain markings mean certain things in geometry.

The figure in part (a) appears to be a square, but based on its markings and the definition of a square, we cannot conclude that. The markings show that the sides of the quadrilateral are equal in length, but equal sides are not enough to make a square. To be a square it also needs four right angles. They angles in the drawing look like right angles, but maybe they are not quite 90°. They could be 89° and 91°, so without the appropriate markings or other information, we cannot assume the angles are right angles. This quadrilateral with four sides of equal length is called a **rhombus**.

Part (b) shows us two types of markings. The small box in the corner of the triangle tells us it is a right angle (measures 90°), so this is a right triangle. We know that the markings on the sides mean that the sides are the same length. A triangle with two sides that are the same length is called an isosceles triangle. Putting both of these facts together, we can label this figure an **isosceles right triangle**.

The arrowheads on the two sides of the quadrilateral of part (c) tells us that those sides are parallel. One pair of parallel sides makes this figure a **trapezoid**.

Example 3

Suppose we cut out the three shapes shown in the last example and place them into a bag. If we reach into the bag and randomly pull out a figure without looking, what is the probability that the shape is a triangle? What is the probability the shape has at least two sides of equal length? What is the probability that the shape has more than four sides?

To calculate probability, we count the number of ways a desired outcome can happen (successes) and divide that by the total number of possible outcomes. This explains why the probability of flipping tails with a fair coin is $\frac{1}{2}$. The number of ways we can get tails is one since there is only one tail, and the total number of outcomes is two (either heads or tails).

In our example, to calculate the probability that we pull out a triangle, we need to count the number of triangles in the bag (one) and divide that by the total number of shapes in the bag (three). This means the probability that we randomly pull out a triangle is $\frac{1}{3}$. To calculate the probability that we pull out a shape with at least two sides of equal length, we first count the number of shapes that would be a success (i.e., would fit this condition). The figures in parts (a) and (b) have at least two sides of equal length, so there are two ways to be successful. When we reach into the bag, there are three possible shapes we could pull out, so the total number of outcomes is three. Therefore, the probability of pulling out a shape with at least two sides of equal length is $\frac{2}{3}$. The probability that we reach into the bag and pull out a shape with more than four sides is calculated in the same way. We know that there are three outcomes (shapes), so three is the denominator. But how many ways can we be successful? Are there any shapes with more than four sides? No, so there are zero ways to be successful. Therefore the probability that we pull out a shape with more than four sides is $\frac{0}{3} = 0$.

Problems

Place the shapes from your Shapes Toolkit into the appropriate regions on the Venn diagram at right. The conditions that the shapes must meet to be placed in each circle are listed in each problem. Note: Create a new Venn diagram for each problem.

#1   #2
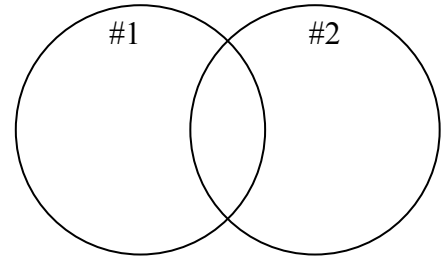
1.   Circle #1: Has more than three sides; Circle #2: Has at least one pair of parallel sides.

2.   Circle #1: Has fewer than four sides; Circle #2: Has at least two sides equal in length.

3.   Circle #1: Has at least one curve; Circle #2: Has at least one obtuse angle.

Each shape below is missing markings. Add the correct markings so that the shape represents the term listed. Note: the pictures may not be drawn to scale.
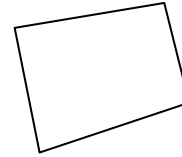
4.   A rectangle.
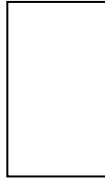
5.   A scalene trapezoid.
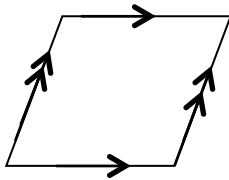
6.   An isosceles right triangle.
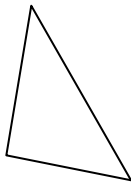
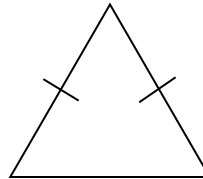7.   An equilateral quadrilateral.

Based on the markings, name the figure below with the most specific name. Note: The figures are not drawn to scale.
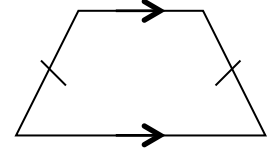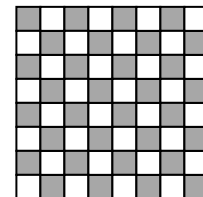
8.

9.

10.

11.   On a spinner there are the numbers 1 through 36 along with 0 and 00. What is the probability that the spinner will stop on the number 17?

12.   When Davis was finished with his checkerboard, he decided to turn it into a dartboard. If he is guaranteed to hit the board, but his dart will hit it randomly, what is the probability he will hit a shaded square?

Answers

1. Common to both circles and placed in the overlapping region are: square, rectangle, parallelogram, isosceles trapezoid, trapezoid, rhombus, and regular hexagon

   Only in Circle #1: quadrilateral, kite, and regular pentagon

   Only in Circle #2: none

   Outside of both circles: circle, scalene triangle, equilateral triangle, isosceles right triangle, isosceles triangle, and scalene right triangle

2. Common to both circles and placed in the overlapping region are: equilateral triangle, isosceles triangle, and isosceles right triangle

   Only in Circle #1: scalene triangle and scalene right triangle

   Only in Circle #2: square, rectangle, parallelogram, rhombus, kite, regular pentagon, isosceles trapezoid, and regular hexagon

   Outside of both circles: circle, quadrilateral, and trapezoid

3. There are no shapes with both characteristics, so there is nothing listed in the overlapping region.

   Only in Circle #1: circle

   Only in Circle #2: scalene triangle, parallelogram, isosceles trapezoid, trapezoid, quadrilateral, kite, rhombus, regular pentagon, and regular hexagon

   Outside of both circles: equilateral triangle, isosceles right triangle, isosceles triangle, scalene right triangle, rectangle, and square

4.   5.   6.   7.
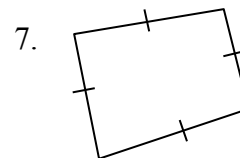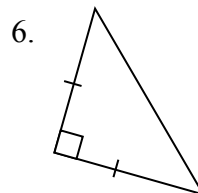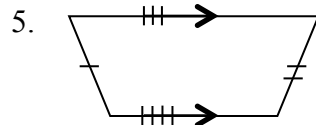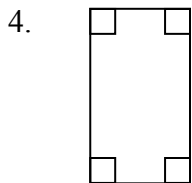
8. A parallelogram

9. An isosceles triangle

10. An isosceles trapezoid

11. P(stop on 17) $= \frac{1}{38}$

12. P(hit a shaded square) $= \frac{1}{2}$