By asking questions such as “What happens if…?” and “What if I change …?” and answering them by trying different things, we can learn quite a lot about different shapes. In the first five lessons of Chapter 1, we explore symmetry, making predictions, perimeter, area, logical arguments, and angles by investigating each of these topics with interesting problems. These five lessons are introductory and help the teacher determine students’ prior knowledge and preview some of the ideas that will be studied in this course. The following examples illustrate the geometry ideas in this section as well as some of the algebra review topics.

See the Math Notes boxes in Lessons 1.1.1, 1.1.2, 1.1.3, 1.1.4, and 1.1.5 for more information about the topics covered in this section.

Example 1

Suppose the rug in Figure 1 is enlarged as shown.

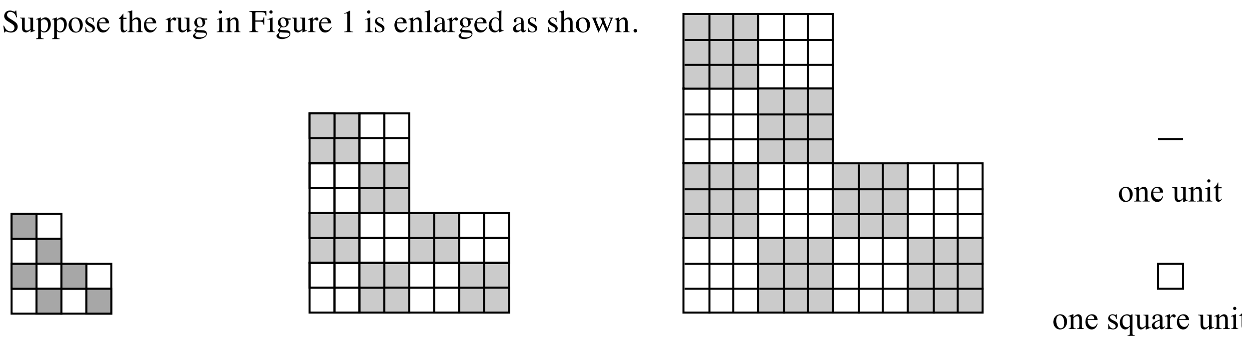


Figure 1 Figure 2 Figure 3

Fill in the table below to show how the perimeter and the area of the rug change as it is enlarged.

Figure Number	1	2	3	4	5	20
Perimeter (units)						
Area (square units)						

Perimeter is the distance (length) around the exterior of a figure on a flat surface while area is the number of non-overlapping square units needed to cover the figure. Perimeter is a unit of length, such as inches or centimeters, while area is measured in square units. Counting the units around the outside of Figure 1, we get a perimeter of 16 units. By counting the number of square units within Figure 1, we get an area of 12 square units. We do the same for the next two figures and record the information in the table.

Figure Number	1	2	3	4	5	20
Perimeter (units)	16	32	48			
Area (square units)	12	48	108			

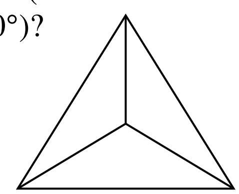
Now comes the task of finding a pattern from these numbers. The perimeters seem to be connected to the number 16, while the areas seem connected to 12. Using this observation, we can rewrite the entries in the table and then extend the pattern to complete it as shown below.

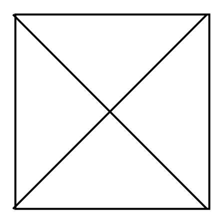
Figure Number	1	2	3	4	5	20
Perimeter (in units)	1(16)	2(16)	3(16)	4(16)	5(16)	20(16)
Area (in square units)	1(12)	4(12)	9(12)	16(12)	25(12)	400(12)

Notice that the multipliers for the areas are the squares of the figure numbers.

Example 2

By using a hinged mirror and a piece of paper, students explored how a kaleidoscope works. The hinge should have been placed so that two edges of the mirror have the same length on the paper, forming an isosceles triangle. The reflection of the triangle in the mirror created shapes with varying numbers of sides. Through this investigation, students saw how angles are related to shapes. In particular, by opening the hinge of the mirror at a certain angle, students could create shapes in the mirror with specific numbers of sides. The hinge represented the angle at the center (or central angle) of the shape. (See Lesson 1.1.5 in the student text.) How many sides would the resulting shape have if the mirror was placed (1) At an obtuse angle (between 90° and 180°)? (2) At a right angle (exactly 90°)? (3) At an acute angle (less than 90°)?





If the central angle is obtuse, the resulting figure is a triangle, so figures formed with this kind of angle are limited to three sides. If the hinge completely opens, it forms a straight angle (measuring 180°), and the figure is no longer a closed shape, but a line. As the hinge closes and forms a right angle, the figure adds another side, creating a quadrilateral As the hinge closes even further, the angle it makes is now acute. As the angle decreases in size this will create more sides on the polygon. It is possible to create a pentagon (five-sided figure), a hexagon (six-sided figure), and, in fact, any number of sides using acute angles of decreasing measures.

Example 3

Solve the equation for x: $2(x-4)+3(x+1)=43+x$

In solving equations such as the one above, we use the Distributive Property to simplify, combine like terms, and isolate the variables on one side of the equal sign and the constant terms (numbers) on the other side.

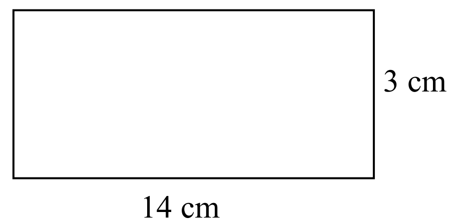
$$2(x-4)+3(x+1)=43+x \quad \text{Distribute}$$
$$2x-8+3x+3=43+x$$
$$5x-5=43+x \quad \text{Simplify}$$
$$4x=48 \quad \text{Divide by 4}$$
$$\frac{4x}{4}=\frac{48}{4}$$
$$x=12$$

See the Math Notes box in Lesson 1.1.4 for another example.

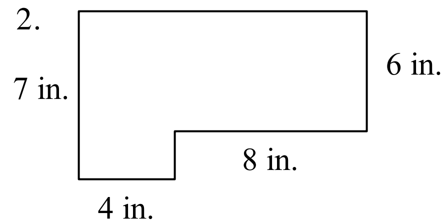
Problems

Find the perimeter and area of each figure below.

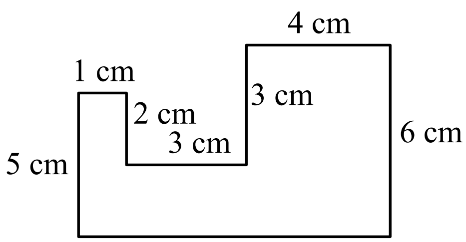
1.



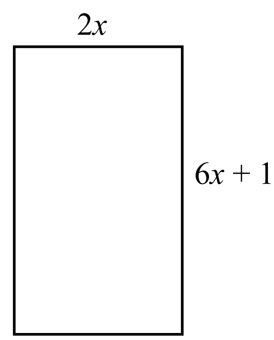
2.



3.



4.

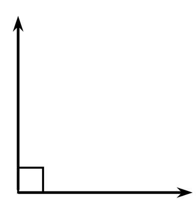


5. If the perimeter for the rectangle in problem 4 is 34 units, write an equation and solve for x.

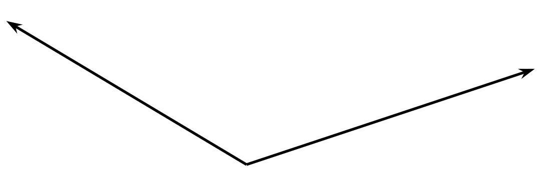
6. Solve for x. Show the steps leading to your solution. $-2x + 6 = 5x - 8$

7. Solve for x. Show the steps leading to your solution. $3(2x - 1) + 9 = 4(x + 3)$

For problems 8-11, estimate the size of each angle to the nearest 10°.
A right angle is shown for reference so you should not need a protractor.
Then classify each angle as either acute, right, obtuse, straight, or circular.



8.



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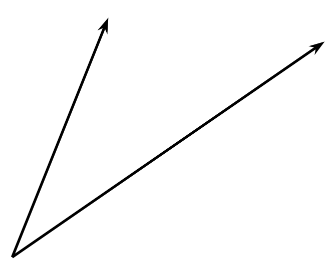
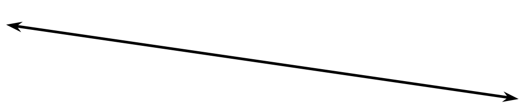
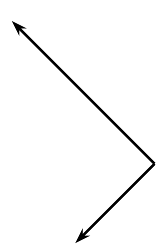


10.



11.



Answers

1. Perimeter = 34 cm, Area = 42 square cm
2. Perimeter = 38 in., Area = 76 sq in.
3. Perimeter = 32 cm, Area = 38 square cm
4. Perimeter = $16x+2$ units, Area = $2x(6x+1)$ or $12x^2+2x$ units^2
5. $2(2x)+2(6x+1)=34$, $x=2$
6. $x=2$
7. $x=3$
8. ≈ 160°, obtuse
9. ≈ 40°, acute
10. 180°, straight
11. 90°, right