

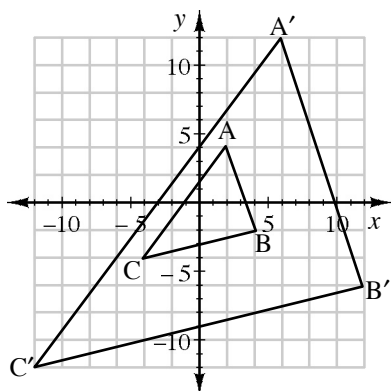
So far, students have measured, described, and transformed geometric shapes. In this chapter we focus on comparing geometric shapes. We begin by dilating shapes: enlarging them as one might on a copy machine. When students compare the original and enlarged shapes closely, they discover that the shape of the figure remains exactly the same (this means the angle measures of the enlarged figure are equal to those of the original figure), but the size changes (the lengths of the sides increase). Although the size changes, the lengths of the corresponding sides all have a constant ratio, known as the zoom factor, or ratio of similarity.

See the Math Notes boxes in Lessons 3.1.1, 3.1.2, 3.1.3, and 3.1.4 for more information about dilations and similar figures.

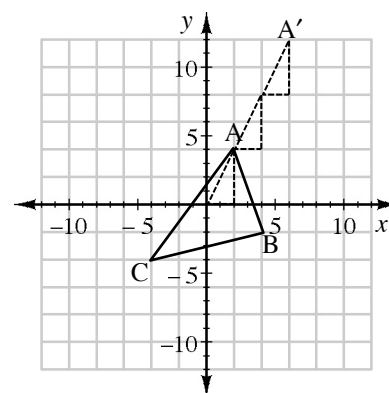
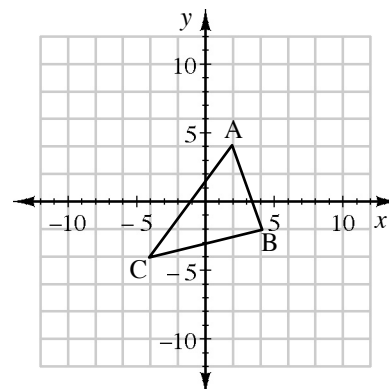
Example 1

Enlarge the figure at right from the origin by a factor of 3.

Students used rubber bands to create a dilation (enlargement) of several shapes. We can do this using a grid and slope triangles. Create a right triangle so that the segment from the origin to point A, (2, 4), is the hypotenuse, one leg lies on the positive x -axis, and the other connects point A to the endpoint of the leg at (2, 0). This triangle is called a slope triangle since it represents the slope of the hypotenuse from (0, 0) to vertex A. Add two more slope triangles exactly like this one along the line from (0, 0) to point A as shown in the figure at right. Using three triangles creates an enlargement by a factor of 3 and gives us the new point A' at (6, 12). Repeat this process for the other two vertices, forming a new slope triangle for each vertex.

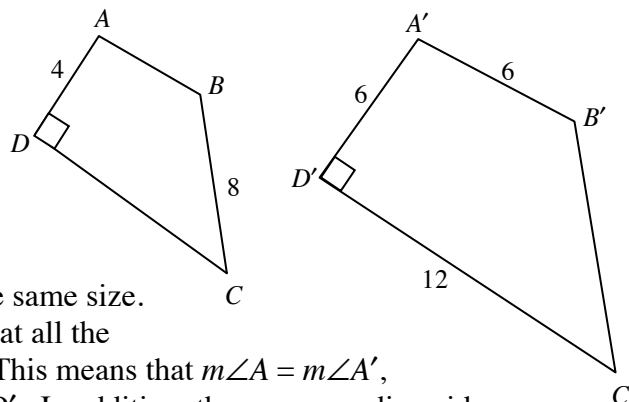


This will give us new points B' at (12, -6) and C' at (-12, -12). Connecting points A' , B' , and C' , we form a new triangle that is an enlargement of the original triangle by a factor of 3, as shown at left.



Example 2

The two quadrilaterals at right are similar. What parts are equal? Can you determine the lengths of any other sides?



Similar figures have the same shape, but not the same size.

Since the quadrilaterals are similar, we know that all the

corresponding angles have the same measure. This means that $m\angle A = m\angle A'$,

$m\angle B = m\angle B'$, $m\angle C = m\angle C'$, and $m\angle D = m\angle D'$. In addition, the corresponding sides are

proportional, which means the ratio of corresponding sides is a constant. To find the ratio, we need to know the lengths of one pair of corresponding sides. From the picture we see that \overline{AD} corresponds to $\overline{A'D'}$. Since these sides correspond, we can write $\frac{AD}{A'D'} = \frac{4}{6}$.

Therefore, the ratio of similarity is $\frac{4}{6}$, or $\frac{2}{3}$. We can use this value to find the lengths of other sides when we know at least one length of a corresponding pair of sides.

$$\frac{AB}{A'B'} = \frac{4}{6}$$

$$\frac{AB}{6} = \frac{4}{6}$$

$$AB = 4$$

$$\frac{BC}{B'C'} = \frac{4}{6}$$

$$\frac{8}{B'C'} = \frac{4}{6}$$

$$4B'C' = 48$$

$$B'C' = 12$$

$$\frac{CD}{C'D'} = \frac{4}{6}$$

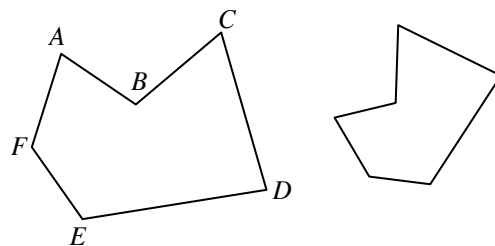
$$\frac{CD}{12} = \frac{4}{6}$$

$$6CD = 48$$

$$CD = 8$$

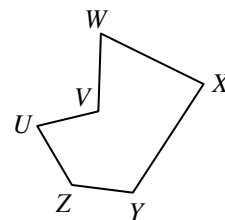
Example 3

The pair of shapes at right is similar ($ABCDEF \sim UVWXYZ$). Label the second figure correctly to reflect the similarity statement.



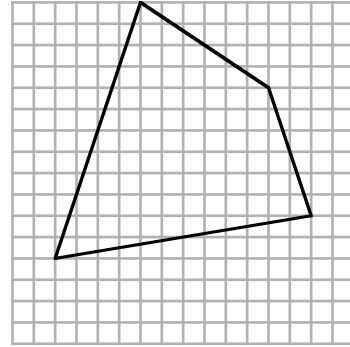
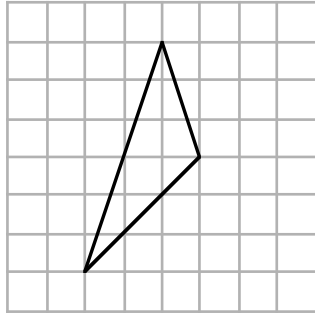
Since similar figures have the same shape, just different sizes, this means that the corresponding angles have equal measure. When we write a similarity statement, we write the letters so that the corresponding angles match up. By the similarity statement, we must have $m\angle A = m\angle U$, $m\angle B = m\angle V$, $m\angle C = m\angle W$, $m\angle D = m\angle X$, $m\angle E = m\angle Y$, and $m\angle F = m\angle Z$.

The smaller figure is labeled at right. If it is difficult to tell which original angle corresponds to its enlargement or reduction, try rotating the figures so that they have the same orientation.



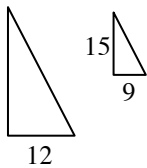
Problems

- Copy the figure below onto graph paper and then enlarge it by a factor of 2.
- Create a figure similar to the one below with a zoom factor of 0.5.

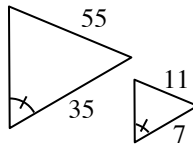


For each pair of similar figures below, find the ratio of similarity, for large:small.

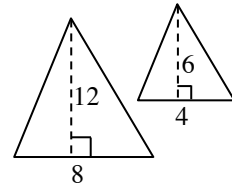
3.



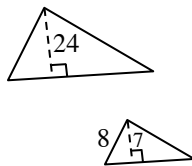
4.



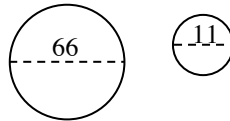
5.



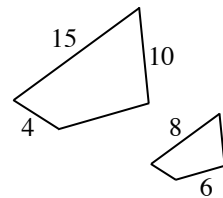
6.



7.

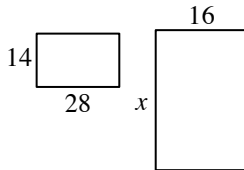


8.

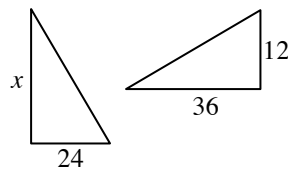


For each pair of similar figures, state the ratio of similarity, then use it to find x .

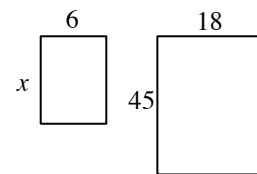
9.



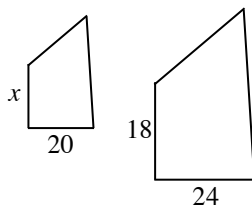
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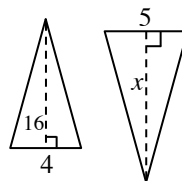
11.



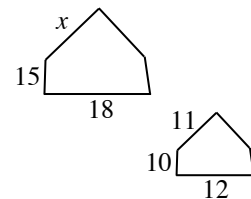
12.



13.



14.



For problems 15 through 20, use the given information and the figure to find each length.

15. $JM = 14, MK = 7, JN = 10$ Find NL .

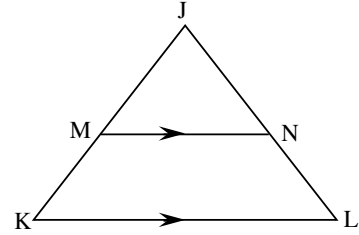
16. $MN = 5, JN = 4, JL = 10$ Find KL .

17. $KL = 10, MK = 2, JM = 6$ Find MN .

18. $MN = 5, KL = 10, JN = 7$ Find JL .

19. $JN = 3, NL = 7, JM = 5$ Find JK .

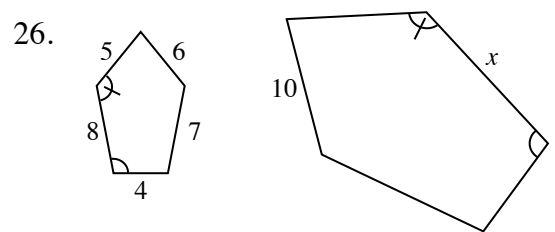
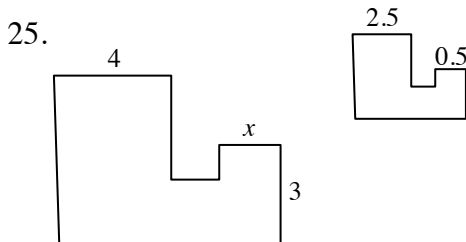
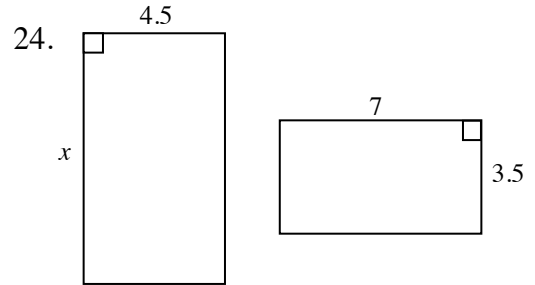
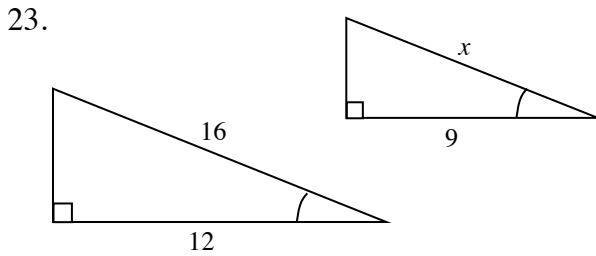
20. $JK = 37, NL = 7, JM = 30$ Find JN .



21. Standing 4 feet from a mirror lying on the flat ground, Palmer, whose eye height is 5 feet, 9 inches, can see the reflection of the top of a tree. He measures the mirror to be 24 feet from the tree. How tall is the tree?

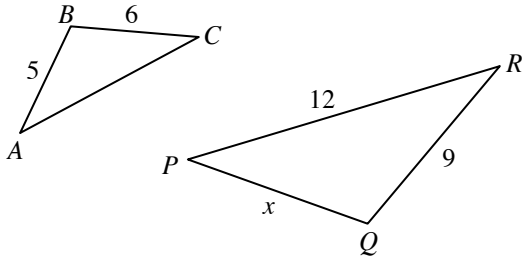
22. The shadow of a statue is 20 feet long, while the shadow of a student is 4 ft long. If the student is 6 ft tall, how tall is the statue?

Each pair of figures below is similar. Use what you know about similarity to solve for x .

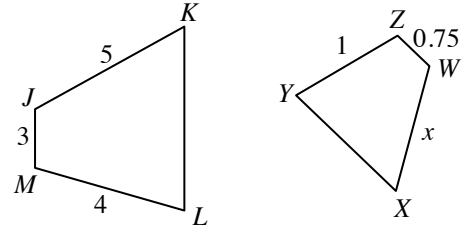


Solve for the missing lengths in the pairs of similar figures below.

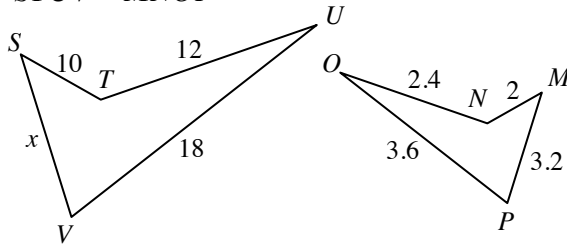
27. $\triangle ABC \sim \triangle PQR$



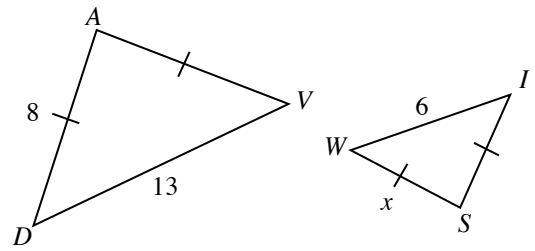
28. $JKLM \sim WXYZ$



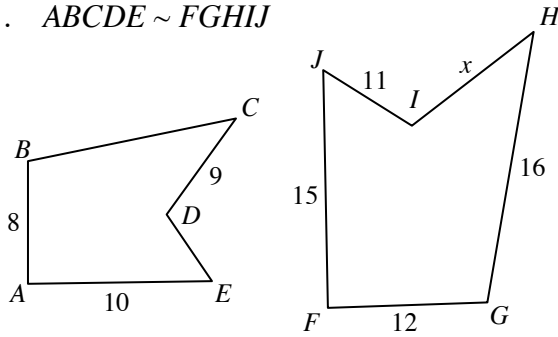
29. $STUV \sim MNOP$



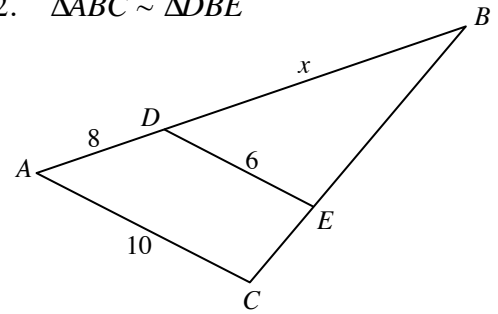
30. $\triangle DAV \sim \triangle ISW$



31. $ABCDE \sim FGHIJ$

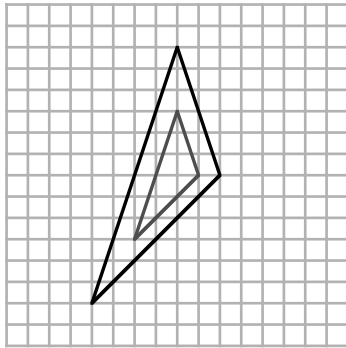


32. $\triangle ABC \sim \triangle DBE$

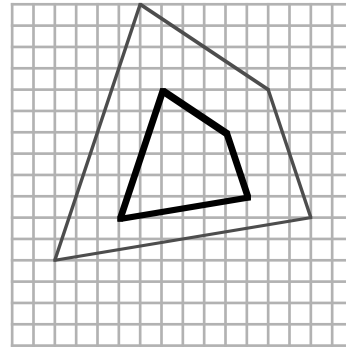


Answers

1.



2.



- | | | | |
|---------------------------|---------------------------|---------------------------|--------------------------------------|
| 3. $\frac{4}{3}$ | 4. $\frac{5}{1}$ | 5. $\frac{2}{1}$ | 6. $\frac{24}{7}$ |
| 7. $\frac{6}{1}$ | 8. $\frac{15}{8}$ | 9. $\frac{7}{8}; x = 32$ | 10. $\frac{2}{1}; x = 72$ |
| 11. $\frac{1}{3}; x = 15$ | 12. $\frac{5}{6}; x = 15$ | 13. $\frac{4}{5}; x = 20$ | 14. $\frac{3}{2}; x = 16.5$ |
| 15. 5 | 16. 12.5 | 17. 7.5 | 18. 14 |
| 19. $16\frac{2}{3}$ | 20. 30 | 21. 34.5 ft | 22. 30 ft |
| 23. $x = 12$ | 24. $x = 9$ | 25. $x = 0.8$ | 26. $x = \frac{40}{3} \approx 13.33$ |
| 27. $x = 7.5$ | 28. $x = 1.25$ | 29. $x = 16$ | 30. $x \approx 3.69$ |
| 31. $x = 13.5$ | 32. $x = 12$ | | |