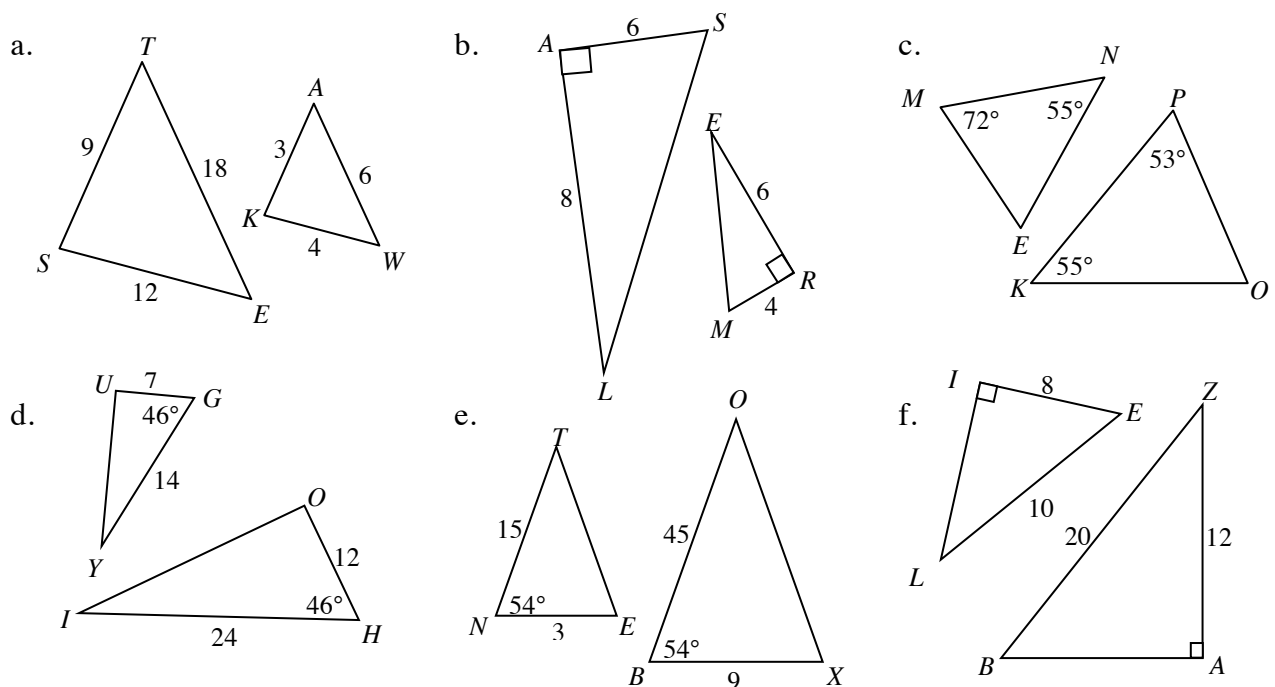


When two figures are related by a series of transformations (including dilations), they are similar. Another way to check for similarity is to measure all the angles and sides of two figures. In this section students develop conditions to shorten the process. These are the **AA Triangle Similarity Condition (AA~)**, the **SAS Triangle Similarity Condition (SAS~)**, and the **SSS Triangle Similarity Condition (SSS~)**. The first condition states that if two pairs of corresponding angles have equal measures, then the triangles are similar. The second condition states that if two pairs of corresponding side lengths have the same ratio, *and* their included angles have the same measure, then the triangles are similar. The third condition states that if all three pairs of corresponding side lengths have the same ratio, then the triangles are similar. Additionally, students found that if similar figures have a ratio of similarity of 1, then the shapes are **congruent**, that is, they have the same size and shape. Students used flowcharts in this section to help organize their information and make logical conclusions about similar triangles. Now students are able to use similar triangles to find side lengths, perimeters, heights, and other measurements.

See the Math Notes boxes in Lessons 3.2.1, 3.2.2, 3.2.4, and 3.2.5 for more information about similar triangles, congruent triangles, and writing flowcharts.

Example 1

Based on the given information, is each pair of triangles similar? If they are similar, write the similarity statement. Justify your answer completely.



We will use the three similarity conditions to test whether or not the triangles are similar.

In part (a), we have the lengths of the three sides, so it makes sense to check whether the SSS~ holds true. Write the ratios of the corresponding side lengths and compare them to see if they are the same, as shown at right. Each ratio reduces to 3, so they are equal. Therefore, $\triangle TES \sim \triangle AWK$ by SSS~.

$$\frac{ST}{KA} \stackrel{?}{=} \frac{TE}{AW} \stackrel{?}{=} \frac{ES}{WK}$$

$$\frac{9}{3} = \frac{18}{6} = \frac{12}{4}$$

The measurements given in part (b) suggest we look at SAS~. $\angle A$ and $\angle R$ are the included angles. Since they are both right angles, they have equal measures. Now we need to check that the corresponding sides lengths have the same ratio, as shown at right.

$$\frac{LA}{ER} \stackrel{?}{=} \frac{AS}{RM}$$

$$\frac{8}{6} \stackrel{?}{=} \frac{6}{4}$$

$$\frac{4}{3} \neq \frac{3}{2}$$

Although the triangles display the SAS~ pattern and the included angles have equal measures, the triangles are not similar because the corresponding side lengths do not have the same ratio.

In part (c), we are given the measures of two angles of each triangle, but not corresponding angles. $m\angle K = 55^\circ = m\angle N$ which is one pair of corresponding angles. For AA~, we need two pairs of equal angles. If we use the fact that the measures of the three angles of a triangle add up to 180° , we can find the measures of $\angle O$ and $\angle E$ as shown at right. Now we see that all pairs of corresponding angles have equal measures, so $\triangle POK \sim \triangle EMN$ by AA~.

$$m\angle O = 180^\circ - 53^\circ - 55^\circ$$

$$m\angle O = 72^\circ$$

$$m\angle E = 180^\circ - 55^\circ - 72^\circ$$

$$m\angle E = 53^\circ$$

Part (d) shows the SAS~ pattern and we can see that the included angles have equal measures, $m\angle G = m\angle H$. We also need to have the ratio of the corresponding side lengths to be equal. Since the two fractions are equal (the second reduces to the first), the corresponding side lengths have the same ratio. Therefore, $\triangle YUG \sim \triangle OH$ by SAS~.

$$\frac{UG}{OH} \stackrel{?}{=} \frac{GY}{HI}$$

$$\frac{7}{12} = \frac{14}{24}$$

In part (e), we see that the included angles have equal measures, $m\angle B = m\angle N$.

Since $\frac{45}{15} = \frac{9}{3} = \frac{3}{1}$, the corresponding sides are proportional. Therefore, $\triangle BOX \sim \triangle NTE$ by SAS~.

In part (f), we only have one pair of angles that are equal (the right angles), but those angles are not between the sides with known lengths. However, we can find the lengths of the third sides by using the Pythagorean Theorem.

$$8^2 + (IL)^2 = 10^2$$

$$64 + (IL)^2 = 100$$

$$(IL)^2 = 36$$

$$IL = 6$$

$$12^2 + (AB)^2 = 20^2$$

$$144 + (AB)^2 = 400$$

$$(AB)^2 = 256$$

$$AB = 16$$

Now that we know all three sides, we can check to see if the triangles are similar by SSS~. Since the ratios of the corresponding sides are the same, $\triangle ELI \sim \triangle BZA$ by SSS~.

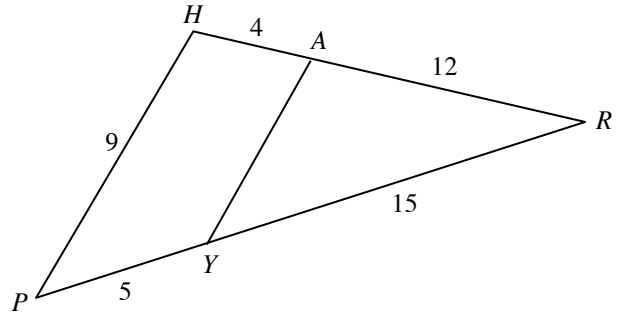
$$\frac{12}{6} \stackrel{?}{=} \frac{16}{8} \stackrel{?}{=} \frac{20}{10}$$

$$2 = 2 = 2$$

Example 2

In the figure at right, $\overline{AY} \parallel \overline{HP}$. Decide whether or not there are any similar triangles in the figure. Justify your answer with a flowchart.

Can you find the length of \overline{AY} ? If so, find it. Justify your answer.



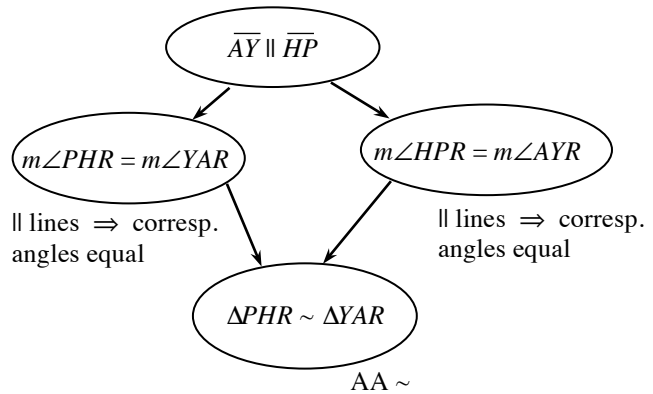
Recalling information we studied in earlier chapters, the parallel lines give us angles with equal measures. In this figure, we have two pairs of corresponding angles with equal measures: $m\angle PHR = m\angle YAR$ and $m\angle HPR = m\angle AYR$. Because two pairs of corresponding angles have equal measures, we can say the triangles are similar: $\triangle PHR \sim \triangle YAR$ by AA \sim . Since the triangles are similar, the lengths of corresponding sides are proportional (i.e., have the same ratio). This means we can write the solution at right.

$$\frac{RA}{RH} = \frac{AY}{HP}$$

$$\frac{12}{16} = \frac{AY}{9}$$

$$AY = \frac{9 \cdot 12}{16} = 6.75$$

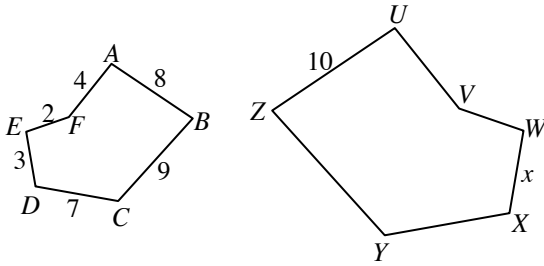
We can justify this result with a flowchart as well. The flowchart at right organizes and states what is written above.



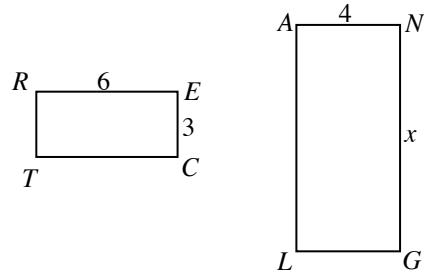
Problems

Each pair of figures below is similar. Write a correct similarity statement and solve for x .

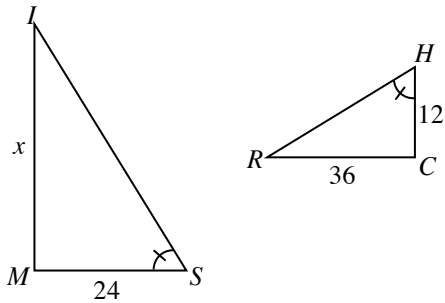
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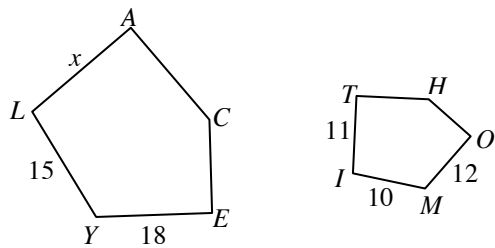
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3.

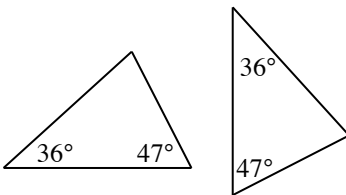


4.

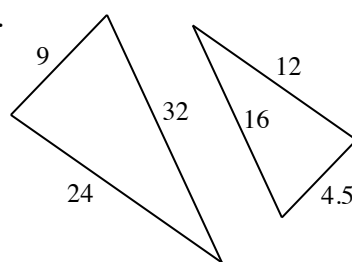


Determine if each pair of triangles is similar. If they are similar, justify your answer.

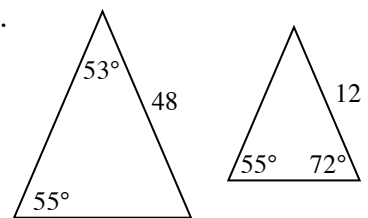
5.



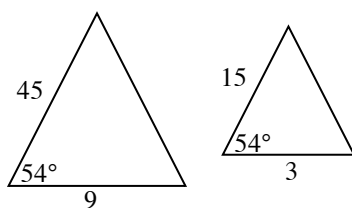
6.



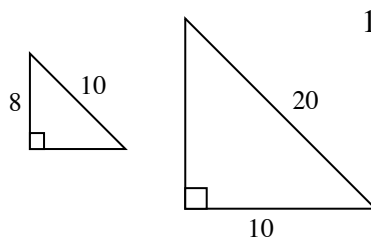
7.



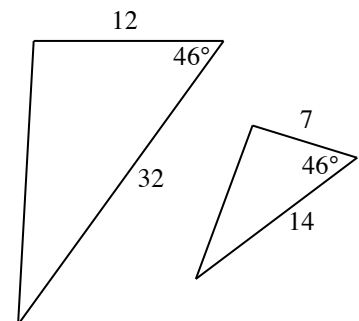
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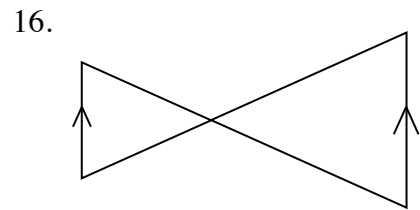
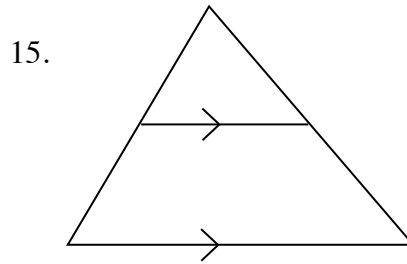
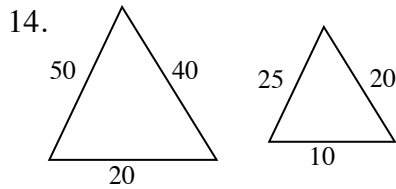
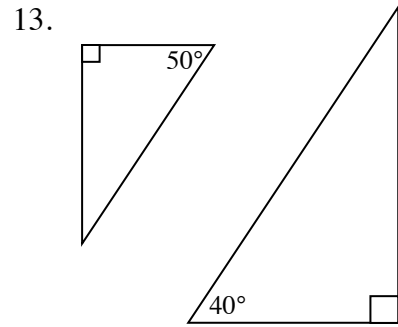
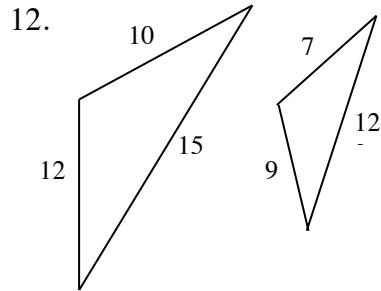
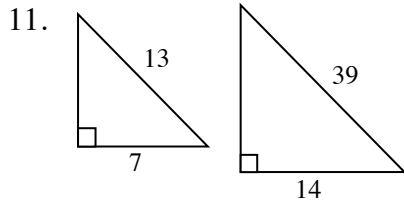


9.

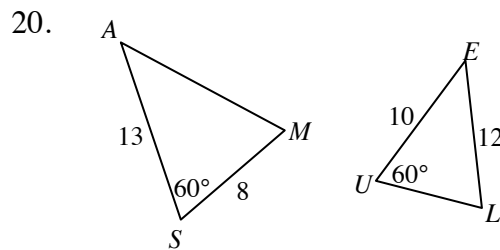
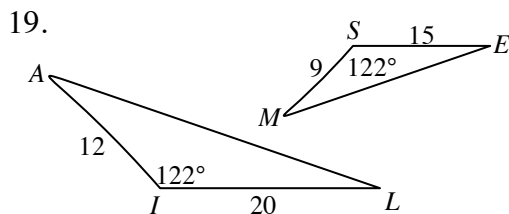
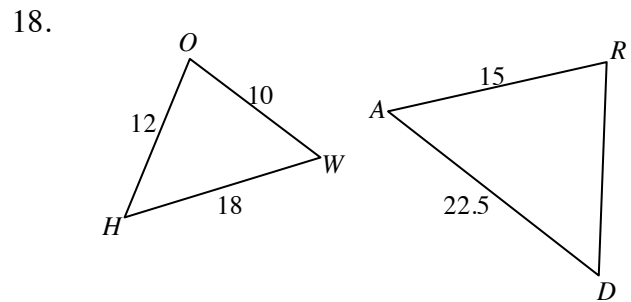
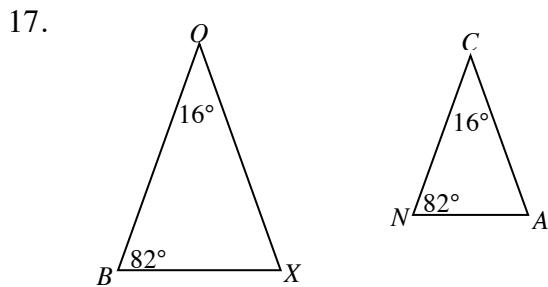


10.

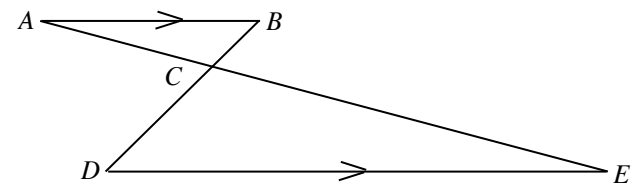




Decide if each pair of triangles is similar. If they are similar, write a correct similarity statement and justify your answer.



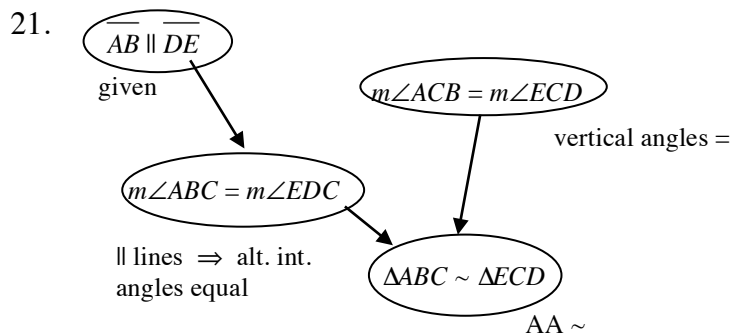
21. In the figure at right $\overline{AB} \parallel \overline{DE}$. Is $\triangle ABC$ similar to $\triangle EDC$? Use a flowchart to organize and justify your answer.



22. Standing four feet from a mirror resting on the flat ground, Palmer, whose eye height is 5 feet, 9 inches, can see the reflection of the top of a tree. He measures the mirror to be 24 feet from the tree. How tall is the tree? Draw a picture to help solve the problem.

Answers

1. $ABCDEF \sim UZYXWV, x = 3.75$
2. $RECT \sim NGLA, x = 8$
3. $\triangle IMS \sim \triangle RCH, x = 72$
4. $LACEY \sim ITHOM, x = 16.5$
5. AA ~
6. SSS ~
7. AA ~
8. SAS ~
9. not ~
10. not ~
11. SAS ~ or SSS ~
12. not ~
13. AA ~
14. SSS ~
15. AA ~
16. AA ~
17. $\triangle BOX \sim \triangle NCA$ by AA ~
18. The triangles are not similar because the sides are not proportional.
 $\frac{12}{15} = \frac{18}{22.5} = 0.8, \frac{10}{13} \approx 0.76$
19. $\triangle ALI \sim \triangle MES$ by SAS ~
20. The triangles are not similar. On $\triangle SAM$, the 60° is included between the two given sides, but on $\triangle UEL$ the angle is not included.



Note: There is more than one way to solve this problem. Corresponding angles could have been used twice rather than mentioning vertical angles.

22. The figures at right show a sketch of the situation and how it translates into a diagram with triangles. $\triangle PFM \sim \triangle TRM$ by AA ~. The proportion is:

$$\frac{x}{5.75} = \frac{24}{4}$$

$$4x = 138$$

$$x = 34.5$$

Therefore, the tree is 34.5 feet tall.

