ANGLES 2.1.1 – 2.1.5

Applications of geometry in "everyday" settings often involve the measures of angles. In this chapter we begin our study of angle measurement. After describing angles and recognizing their characteristics, students complete an Angle Relationships Toolkit (Lesson 2.1.3 Resource Page). The toolkit lists some special angles and then students record important information about them. The list includes vertical angles (which are always equal in measure), straight angles (which measure 180°), corresponding angles, alternate interior angles, and same-side interior angles.

See the Math Notes boxes in Lessons 2.1.1 and 2.1.4 for more information about angle relationships.

Example 1

In each figure below, find the measures of angles *a*, *b*, and/or *c*. Justify your answers.

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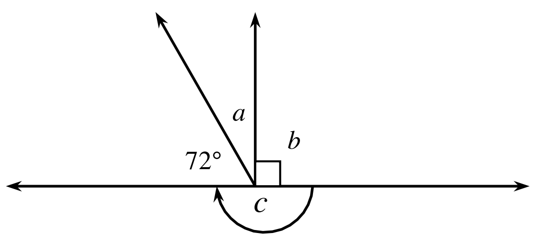

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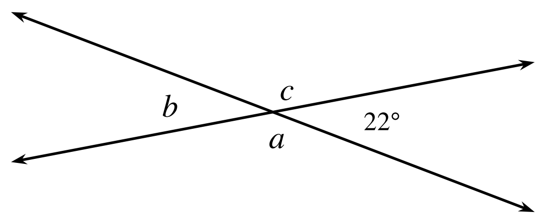

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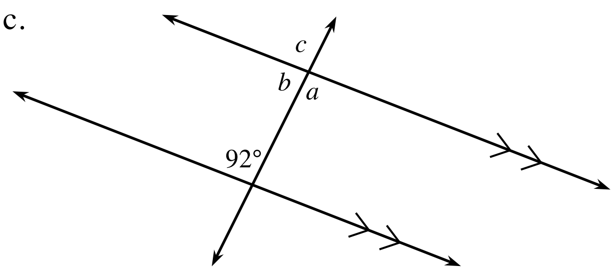

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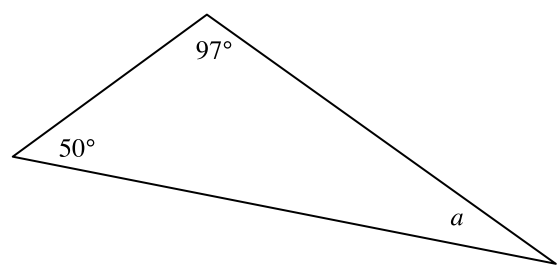

Each figure gives us information that enables us to find the measures of the other angles. In part (a), the little box at angle *b* tells us that angle *b* is a right angle, so $m\angle b = 90^\circ$. The angle labeled *c* is a straight angle (it is opened wide enough to form a straight line) so $m\angle c = 180^\circ$. To calculate $m\angle a$ we need to realize that $\angle a$ and the 72° angle are complementary which means together they sum to 90°. Therefore, $m\angle a + 72^\circ = 90^\circ$ which tells us that $m\angle a = 18^\circ$.

In part (b) we will use two pieces of information, one about supplementary angles and one about vertical angles. First, $m\angle a$ and the 22° angle are supplementary because they form a straight angle (line), so the sum of their measures is 180°. Subtracting from 180° we find that $m\angle a = 158^\circ$. Vertical angles are formed when two lines intersect. They are the two pairs of

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angles that are opposite (across from) each other where the lines cross. Their angle measures are always equal. Since the 22° angle and $\angle b$ are a pair of vertical angles, $m\angle b = 22^\circ$. Similarly, $\angle a$ and $\angle c$ are vertical angles, and therefore equal, so $m\angle a = m\angle c = 158^\circ$.

The figure in part (c) shows two parallel lines that are intersected by a transversal. When this happens we have several pairs of angles with equal measures. $\angle a$ and the 92° angle are called alternate interior angles, and since the lines are parallel (as indicated by the double arrows on the lines), these angles have equal measures. Therefore, $m\angle a = 92^\circ$. There are several ways to calculate the remaining angles. One way is to realize that $\angle a$ and $\angle b$ are supplementary. Another uses the fact that $\angle b$ and the 92° angle are same-side interior angles, which makes them supplementary because the lines are parallel. Either way gives the same result: $m\angle b = 180^\circ - 92^\circ = 88^\circ$. There is also more than one way to calculate $m\angle c$. We know that $\angle c$ and $\angle b$ are supplementary. Alternately, $\angle c$ and the 92° angle are corresponding angles, which are equal because the lines are parallel. A third way is to see that $\angle a$ and $\angle c$ are vertical angles. With any of these approaches, $m\angle c = 92^\circ$.

Part (d) is a triangle. In class, students investigated the measures of the angles in a triangle. They found that the sum of the measures of the three angles always equals 180°. Knowing this, we can calculate $m\angle a$: $m\angle a + 50^\circ + 97^\circ = 180^\circ$. Therefore, $m\angle a = 33^\circ$.

Problems

Use the geometric properties and theorems you have learned to solve for x in each diagram and write the property or theorem you use in each case.

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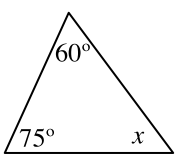

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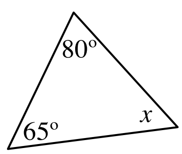

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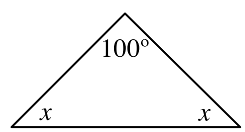

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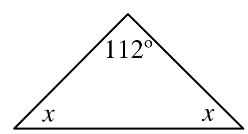

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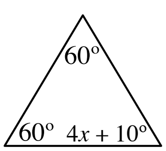

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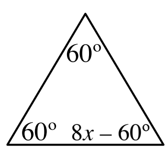

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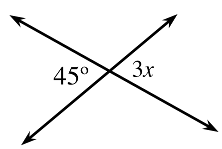

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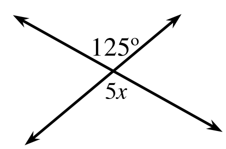

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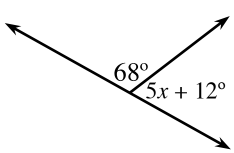

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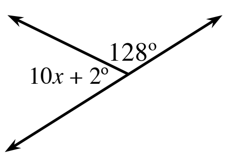

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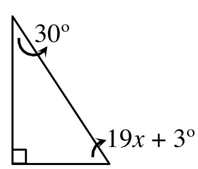

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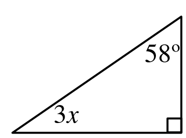

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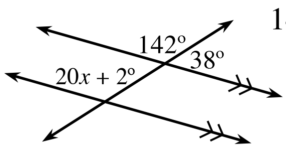

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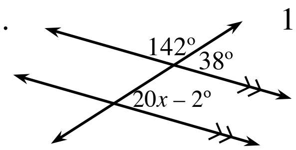

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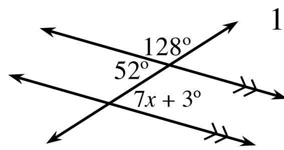

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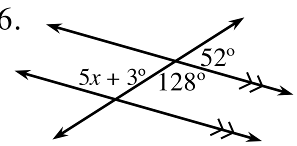

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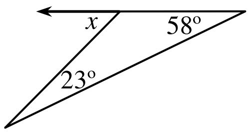

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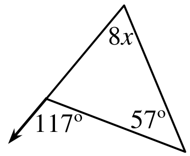

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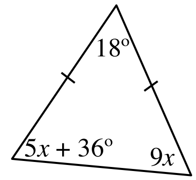

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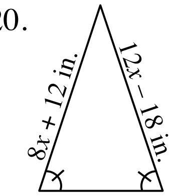

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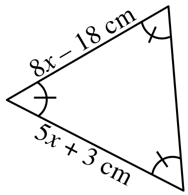

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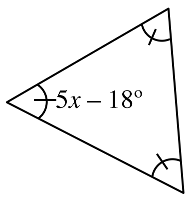

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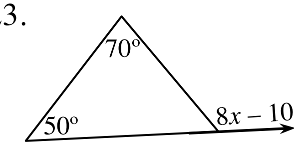

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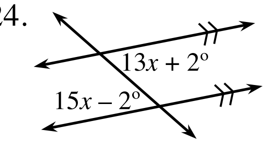

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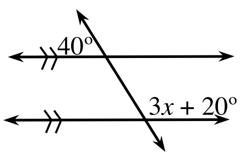

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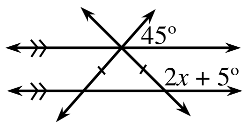

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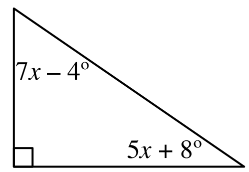

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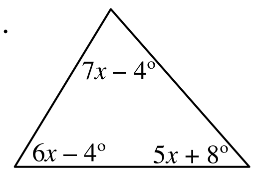

Use what you know about angle measures to find x, y, or z.

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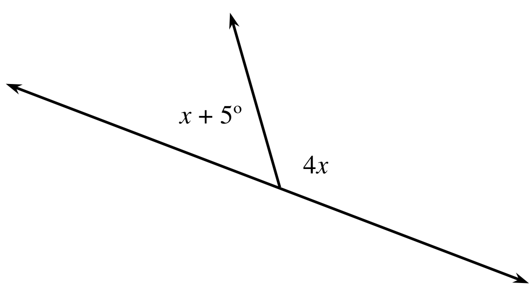

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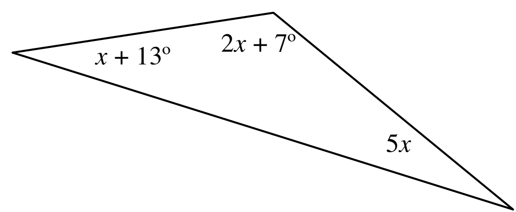

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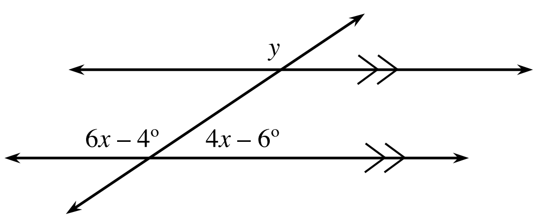

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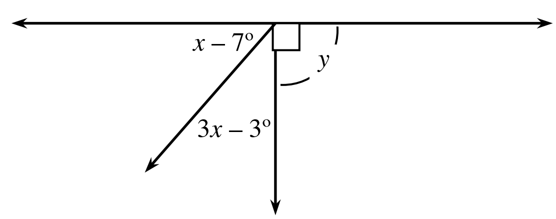

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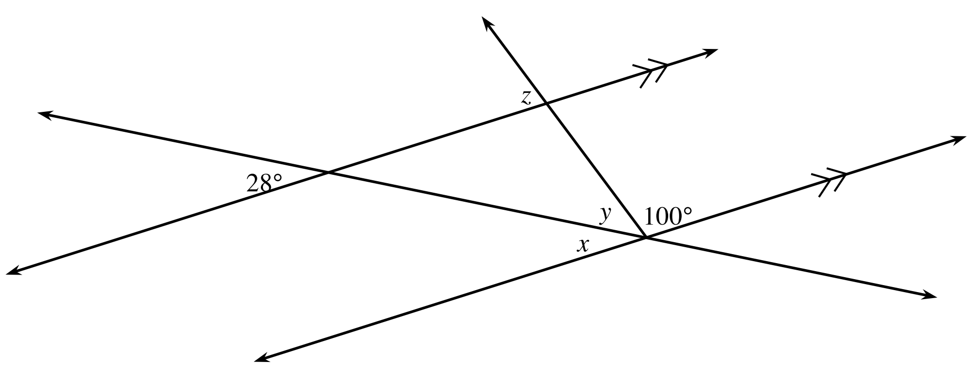

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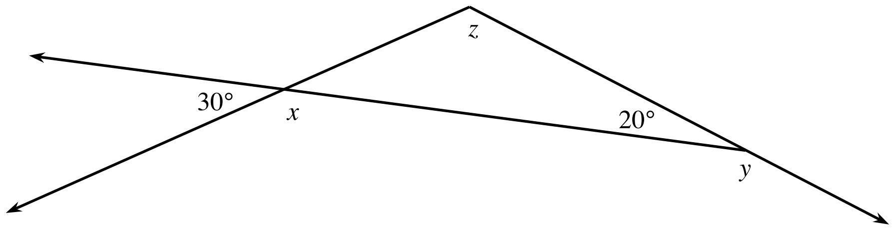


In Lesson 2.1.5 we used what we have learned about angle measures to create proofs by contradiction. (See the Math Notes box in Lesson 2.1.5.) Use this method of proof to justify each of your conclusions to problems 35 and 36 below.

35. Nik scored 40 points lower than Tess on their last math test. The scores could range from 0 to 100 points. Could Tess have scored a 30 on this test? Justify using a proof by contradiction.

36. Can a triangle have two right angles? Justify your answer with a proof by contradiction.

Answers

1. $x = 45°$	2. $x = 35°$	3. $x = 40°$	4. $x = 34°$
5. $x = 12.5°$	6. $x = 15°$	7. $x = 15°$	8. $x = 25°$
9. $x = 20°$	10. $x = 5°$	11. $x = 3°$	12. $x = 10\frac{2}{3}°$
13. $x = 7°$	14. $x = 2°$	15. $x = 7°$	16. $x = 25°$
17. $x = 81°$	18. $x = 7.5°$	19. $x = 9°$	20. $x = 7.5$ in.
21. $x = 7$ cm	22. $x = 15.6°$	23. $x = 16.25°$	24. $x = 2°$
25. $x = 40°$	26. $x = 65°$	27. $x = 7\frac{1}{6}°$	28. $x = 10°$

29. $(x + 5°) + 4x = 180°, x = 35°$

30. $(x + 13°) + (2x + 7°) + 5x = 180°, x = 20°$

31. $(6x - 4°) + (4x - 6°) = 180, x = 19°, y = 110°$

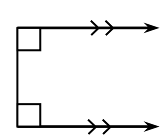
32. $(x - 7°) + (3x - 3°) = 90°, x = 25°, y = 90°$

33. $x = 28°, y = 52°, z = 80°$

34. $x = 150°, y = 160°, z = 130°$

35. If Tess scored 30 points, then Nik's score would be –10, which is impossible. So Tess cannot have a score of 30 points.

36. If a triangle has two right angles, then the measure of the third angle must be zero. However, this is impossible, so a triangle cannot have two right angles. OR: If a triangle has two 90° angles, the two sides that intersect with the side between them would be parallel and never meet to complete the triangle, as shown in the figure.



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